ASHLAWN SCHOOL

A-Level Maths
Y11-Y12 Transition

How to use this document:

- Use the contents page and any blue text to navigate the document.
- Read through the written examples.
- Complete the sets of practice questions.
- Use the corresponding video for each practice set to support and to self-assess.
- Complete additional practice, using the solutions at the end of the document to self-assess.

Contents

Practice Assessment

1. Simplify the following expressions:

a) $2x^2 \times x^3$

b) $\frac{x^6}{x^2}$

c) $\frac{54x^5y^4}{18x^2y^2}$

d) $9x^{-\frac{1}{3}} \times (4x^4)^{\frac{3}{2}}$

2. Solve: $3x^4 = 48$

3. Find the value of x: $5^{x-1} = 125$

4.

a) Simplify $\sqrt{360}$

b) Expand and simplify: $(3 + 5\sqrt{3})(2 - \sqrt{3})$

c) Simplify: $\frac{3+5\sqrt{3}}{2+\sqrt{3}}$

5. Expand and simplify

$(3x + 2)^3$

6. Fully factorise:

a) $x^2 - 49$

b) $x^2 + 8x + 15$

c) $6x^2 + 5x - 6$

7. Solve the following quadratic equations:

a) $x^2 + 3x = 40$

b) $x^2 - 4x = -3$

8. Solve these simultaneous equations:

a)

$x + y = 11$

$3x - y = 1$

b)

$x - 2y = 5$

$4x + y = 3$

c)

$x^2 + y = 7$

$x + y^2 = 11$

9. Solve the inequalities:

a) $6x - 4 \leq 14$

b) $5x + 3 \geq 2x + 12$

c) $x^2 - 10x + 21 > 0$

10. Functions:

Let $f(x) = 4x - 2$ and $g(x) = x^2 + 1$

a) $f(3)$

b) $fg(x)$

c) $Find\ the\ value\ when\ f^{-1}(x) = 5$

Solutions

Extension activities: Read/Watch/Listen

Read	Watch	Listen
A range of articles https://plus.maths.org/content/articles including "Maths in a minute: Invariants" **"Humble Pi" by Matt Parker** – Fun look at real-life maths mistakes.	A range of videos https://plus.maths.org/content/tags/video including "Complex numbers" "Women of mathematics"	**Numberphile Podcast** **Breaking Math Podcast**

Indices

Worked Examples

Simplify $\frac{x^5}{x^2}$

$\frac{x^5}{x^2} = x^3$	use the rule $\frac{a^m}{a^n} = a^{m-n}$ to give $\frac{x^5}{x^2} = x^{5-2} = x^3$

Simplify $6x^6 \times 3x^4$

$6x^6 \times 3x^4 = 18x^2$	$6 \times 3 = 18$ and then use the rule $a^m \times a^n = a^{m+n}$ to give $x^6 \times x^4 = x^{6+4} = x^{10}$

Simplify $(x^4)^2 \times 3x^5$

$(x^4)^2 \times 3x^5 = 3x^{13}$	$3 \times 1 = 3$ and then use the rule $(a^m)^n = a^{mn}$ following by to give $a^m \times a^n = a^{m+n}$ $(x^4)^2 \times x^5 = x^{4\times 2} \times x^5$ $= x^8 \times x^5$ $= x^{8+5}$ $= x^{13}$

Evaluate $9^{\frac{1}{2}}$

$9^{\frac{1}{2}} = \sqrt{9}$ $= 3$	Use the rule $a^{\frac{1}{n}} = \sqrt[n]{a}$

Evaluate $27^{\frac{2}{3}}$

$27^{\frac{2}{3}} = \left(\sqrt[3]{27}\right)^2$ $= 3^2$ $= 9$	**1** Use the rule $a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m$ **2** Use $\sqrt[3]{27} = 3$

Evaluate 4^{-2}

$4^{-2} = \frac{1}{4^2}$ $= \frac{1}{16}$	**1** Use the rule $a^{-m} = \frac{1}{a^m}$ **2** Use $4^2 = 16$

Practice Questions

Set 1

Simplify these expressions:

a. $x^2 \times x^5$ **b.** $2r^2 \times 3r^3$ **c.** $\frac{b^7}{b^4}$ **d.** $6x^5 \div 3x^3$ **e.** $(a^3)^2 \times 2a^2$ **f.** $(3x^2)^3 \div x^4$

Set 2

Simplify:

a. $\frac{x^3}{x^{-3}}$ **b.** $x^{\frac{1}{2}} \times x^{\frac{3}{2}}$ **c.** $(x^3)^{\frac{2}{3}}$ **d.** $2x^{1.5} \div 4x^{-0.25}$ **e.** $\sqrt[3]{125x^6}$ **f.** $\frac{2x^2 - x}{x^5}$

Set 3

Evaluate:

a. $9^{\frac{1}{2}}$ **b.** $64^{\frac{1}{3}}$ **c.** $49^{\frac{3}{2}}$ **d.** $25^{-\frac{3}{2}}$

Video Solutions

Set 1

Set 2

Set 3

Additional Practice

Set 1

1. (a) Simplify $a^4 \times a^5$

(b) Simplify $\frac{45e^6 f^8}{5ef^2}$

(c) Write down the value of $9^{\frac{1}{2}}$

2. (a) Simplify $x^7 \times x^3$

(b) Simplify $(m^4)^3$

(c) Simplify $\frac{36af^8}{12a^5 f^2}$

Set 2

1 Simplify.

a $\dfrac{3x^2 \times x^3}{2x^2}$ **b** $\dfrac{10x^5}{2x^2 \times x}$

c $\dfrac{3x \times 2x^3}{2x^3}$ **d** $\dfrac{7x^3y^2}{14x^5y}$

e $\dfrac{y^2}{y^{\frac{1}{2}} \times y}$ **f** $\dfrac{c^{\frac{1}{2}}}{c^2 \times c^{\frac{3}{2}}}$

g $\dfrac{\left(2x^2\right)^3}{4x^0}$ **h** $\dfrac{x^{\frac{1}{2}} \times x^{\frac{3}{2}}}{x^{-2} \times x^3}$

Set 3

1 Evaluate.

a $49^{\frac{1}{2}}$ **b** $64^{\frac{1}{3}}$

2 Evaluate.

a $25^{\frac{3}{2}}$ **b** $8^{\frac{5}{3}}$

3 Evaluate.

a 5^{-2} **b** 4^{-3}

4 Evaluate.

a $4^{-\frac{1}{2}}$ **b** $27^{-\frac{2}{3}}$

Solutions

Surds

Worked Examples

Simplify $\sqrt{50}$

$\sqrt{50} = \sqrt{25 \times 2}$ $= \sqrt{25} \times \sqrt{2}$ $= 5 \times \sqrt{2}$ $= 5\sqrt{2}$	**1** Choose two numbers that are factors of 50. One of the factors must be a square number **2** Use the rule $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$ **3** Use $\sqrt{25} = 5$

Simplify $\sqrt{147} - 2\sqrt{12}$

$\sqrt{147} - 2\sqrt{12}$ $= \sqrt{49 \times 3} - 2\sqrt{4 \times 3}$ $= \sqrt{49} \times \sqrt{3} - 2\sqrt{4} \times \sqrt{3}$ $= 7 \times \sqrt{3} - 2 \times 2 \times \sqrt{3}$ $= 7\sqrt{3} - 4\sqrt{3}$ $= 3\sqrt{3}$	**1** Simplify $\sqrt{147}$ and $2\sqrt{12}$. Choose two numbers that are factors of 147 and two numbers that are factors of 12. One of each pair of factors must be a square number **2** Use the rule $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$ **3** Use $\sqrt{49} = 7$ and $\sqrt{4} = 2$ **4** Collect like terms

Simplify $(\sqrt{7} + \sqrt{2})(\sqrt{7} - \sqrt{2})$

$(\sqrt{7} + \sqrt{2})(\sqrt{7} - \sqrt{2})$ $= \sqrt{49} - \sqrt{7}\sqrt{2} + \sqrt{2}\sqrt{7} - \sqrt{4}$ $= 7 - 2$ $= 5$	**1** Expand the brackets. A common mistake here is to write $(\sqrt{7})^2 = 49$ **2** Collect like terms: $-\sqrt{7}\sqrt{2} + \sqrt{2}\sqrt{7}$ $= -\sqrt{7}\sqrt{2} + \sqrt{7}\sqrt{2} = 0$

Rationalise $\frac{1}{\sqrt{3}}$

$\frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$ $= \frac{1 \times \sqrt{3}}{\sqrt{9}}$ $= \frac{\sqrt{3}}{3}$	**1** Multiply the numerator and denominator by $\sqrt{3}$ **2** Use $\sqrt{9} = 3$

Rationalise and simplify $\frac{3}{2+\sqrt{5}}$

$\frac{3}{2+\sqrt{5}} = \frac{3}{2+\sqrt{5}} \times \frac{2-\sqrt{5}}{2-\sqrt{5}}$	**1** Multiply the numerator and denominator by $2-\sqrt{5}$
$= \frac{3(2-\sqrt{5})}{(2+\sqrt{5})(2-\sqrt{5})}$	**2** Expand the brackets
$= \frac{6-3\sqrt{5}}{4+2\sqrt{5}-2\sqrt{5}-5}$	**3** Simplify the fraction
$= \frac{6-3\sqrt{5}}{-1}$	**4** Divide the numerator by -1 Remember to change the sign of all terms when dividing by -1
$= 3\sqrt{5}-6$	

Practice Questions

Set 1

Simplify:

a. $\sqrt{12}$ **b.** $\frac{\sqrt{20}}{2}$ **c.** $5\sqrt{6} - 2\sqrt{24} + \sqrt{294}$

Set 2

Expand and simplify if possible:

a. $\sqrt{2}(5 - \sqrt{3})$ **b.** $(2 - \sqrt{3})(5 + \sqrt{3})$

Set 3

Rationalise the denominator of:

a. $\frac{1}{\sqrt{3}}$ **b.** $\frac{1}{3+\sqrt{2}}$ **c.** $\frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}-\sqrt{2}}$ **d.** $\frac{1}{(1-\sqrt{3})^2}$

Video Solutions

Set 1

Set 2

Set 3

Additional Practice

Set 1

1 Simplify.

a $\sqrt{45}$ **b** $\sqrt{125}$

c $\sqrt{48}$ **d** $\sqrt{175}$

2 Simplify.

a $\sqrt{72}+\sqrt{162}$ **b** $\sqrt{45}-2\sqrt{5}$

c $\sqrt{50}-\sqrt{8}$ **d** $\sqrt{75}-\sqrt{48}$

e $2\sqrt{28}+\sqrt{28}$ **f** $2\sqrt{12}-\sqrt{12}+\sqrt{27}$

Set 2

1 Expand and simplify.

a $(\sqrt{2}+\sqrt{3})(\sqrt{2}-\sqrt{3})$ **b** $(3+\sqrt{3})(5-\sqrt{12})$

c $(4-\sqrt{5})(\sqrt{45}+2)$ **d** $(5+\sqrt{2})(6-\sqrt{8})$

2 Expand and simplify $\left(\sqrt{x}+\sqrt{y}\right)\left(\sqrt{x}-\sqrt{y}\right)$

3 Work out the value of $\left(\sqrt{2}+\sqrt{8}\right)^2$

4 Expand $(1+\sqrt{2})(3-\sqrt{2})$

Give your answer in the form $a+b\sqrt{2}$ where a and b are integers.

Set 3

1 Rationalise and simplify, if possible.

a $\frac{1}{\sqrt{5}}$ **b** $\frac{1}{\sqrt{11}}$

c $\frac{2}{\sqrt{7}}$ **d** $\frac{2}{\sqrt{8}}$

e $\frac{2}{\sqrt{2}}$ **f** $\frac{5}{\sqrt{5}}$

2 Rationalise and simplify.

a $\frac{1}{3-\sqrt{5}}$ **b** $\frac{2}{4+\sqrt{3}}$ **c** $\frac{6}{5-\sqrt{2}}$

3 Rationalise and simplify, if possible.

a $\frac{1}{\sqrt{9}-\sqrt{8}}$ **b** $\frac{1}{\sqrt{x}-\sqrt{y}}$

Solutions

Algebraic Methods

Worked Examples

Factorise $x^2 + 3x - 10$

$b = 3$, $ac = -10$ So $x^2 + 3x - 10 = x^2 + 5x - 2x - 10$ $= x(x + 5) - 2(x + 5)$ $= (x + 5)(x - 2)$	**1** Work out the two factors of $ac = -10$ which add to give $b = 3$ (5 and -2) **2** Rewrite the b term ($3x$) using these two factors **3** Factorise the first two terms and the last two terms **4** $(x + 5)$ is a factor of both terms

Factorise $6x^2 - 11x - 10$

$b = -11$, $ac = -60$ So $6x^2 - 11x - 10 = 6x^2 - 15x + 4x - 10$ $= 3x(2x - 5) + 2(2x - 5)$ $= (2x - 5)(3x + 2)$	**1** Work out the two factors of $ac = -60$ which add to give $b = -11$ (-15 and 4) **2** Rewrite the b term ($-11x$) using these two factors **3** Factorise the first two terms and the last two terms **4** $(2x - 5)$ is a factor of both terms

Factorise $4x^2 - 25y^2$

$4x^2 - 25y^2 = (2x + 5y)(2x - 5y)$	This is the difference of two squares as the two terms can be written as $(2x)^2$ and $(5y)^2$

Simplify $\dfrac{x^2-4x-21}{2x^2+9x+9}$

$\dfrac{x^2-4x-21}{2x^2+9x+9}$	**1** Factorise the numerator and the denominator
For the numerator: $b=-4$, $ac=-21$	**2** Work out the two factors of $ac=-21$ which add to give $b=-4$ (-7 and 3)
So $x^2-4x-21=x^2-7x+3x-21$	**3** Rewrite the b term ($-4x$) using these two factors
$=x(x-7)+3(x-7)$	**4** Factorise the first two terms and the last two terms
$=(x-7)(x+3)$	**5** $(x-7)$ is a factor of both terms
For the denominator: $b=9$, $ac=18$	**6** Work out the two factors of $ac=18$ which add to give $b=9$ (6 and 3)
So $2x^2+9x+9=2x^2+6x+3x+9$	**7** Rewrite the b term ($9x$) using these two factors
$=2x(x+3)+3(x+3)$	**8** Factorise the first two terms and the last two terms
$=(x+3)(2x+3)$	**9** $(x+3)$ is a factor of both terms
So $\dfrac{x^2-4x-21}{2x^2+9x+9}=\dfrac{(x-7)(x+3)}{(x+3)(2x+3)}$	**10** $(x+3)$ is a factor of both the numerator and denominator so cancels out as a value divided by itself is 1
$=\dfrac{x-7}{2x+3}$	

Practice Questions

Set 1

Simplify these fractions:

a. $\dfrac{7x^4-2x^3+6x}{x}$ **b.** $\dfrac{(x+7)(2x-1)}{(2x-1)}$ **c.** $\dfrac{x^2+7x+12}{(x+3)}$

d. $\dfrac{x^2+6x+5}{x^2+3x-10}$ **e.** $\dfrac{2x^2+11x+12}{(x+3)(x+4)}$

Video Solutions

Set 1

Additional Practice

Set 1

1 Simplify the algebraic fractions.

a $\dfrac{2x^2+4x}{x^2-x}$ **b** $\dfrac{x^2+3x}{x^2+2x-3}$

c $\dfrac{x^2-2x-8}{x^2-4x}$ **d** $\dfrac{x^2-5x}{x^2-25}$

e $\dfrac{x^2-x-12}{x^2-4x}$ **f** $\dfrac{2x^2+14x}{2x^2+4x-70}$

2 Simplify

a $\dfrac{9x^2-16}{3x^2+17x-28}$ **b** $\dfrac{2x^2-7x-15}{3x^2-17x+10}$

Solutions

Quadratic Equations

Worked Examples

Solve $5x^2 = 15x$

$5x^2 = 15x$	**1** Rearrange the equation so that all of the terms are on one side of the equation and it is equal to zero. Do not divide both sides by x as this would lose the solution $x = 0$.
$5x^2 - 15x = 0$	
$5x(x - 3) = 0$	**2** Factorise the quadratic equation. $5x$ is a common factor.
So $5x = 0$ or $(x - 3) = 0$	**3** When two values multiply to make zero, at least one of the values must be zero.
Therefore $x = 0$ or $x = 3$	**4** Solve these two equations.

Solve $x^2 + 7x + 12 = 0$

$x^2 + 7x + 12 = 0$	**1** Factorise the quadratic equation. Work out the two factors of $ac = 12$ which add to give you $b = 7$. (4 and 3)
$b = 7$, $ac = 12$	
$x^2 + 4x + 3x + 12 = 0$	**2** Rewrite the b term ($7x$) using these two factors.
$x(x + 4) + 3(x + 4) = 0$	**3** Factorise the first two terms and the last two terms.
$(x + 4)(x + 3) = 0$	**4** $(x + 4)$ is a factor of both terms.
So $(x + 4) = 0$ or $(x + 3) = 0$	**5** When two values multiply to make zero, at least one of the values must be zero.
Therefore $x = -4$ or $x = -3$	**6** Solve these two equations.

Solve $3x^2 - 7x - 2 = 0$. Give your solutions in surd form.

$a = 3$, $b = -7$, $c = -2$	**1** Identify a, b and c, making sure you get the signs right and write down the formula.
$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	Remember that $-b \pm \sqrt{b^2 - 4ac}$ is all over $2a$, not just part of it.
$x = \dfrac{-(-7) \pm \sqrt{(-7)^2 - 4(3)(-2)}}{2(3)}$	**2** Substitute $a = 3$, $b = -7$, $c = -2$ into the formula.
$x = \dfrac{7 \pm \sqrt{73}}{6}$	**3** Simplify. The denominator is 6 when $a = 3$. A common mistake is to always write a denominator of 2.
So $x = \dfrac{7 - \sqrt{73}}{6}$ or $x = \dfrac{7 + \sqrt{73}}{6}$	**4** Write down both the solutions.

Complete the square for the expression $x^2 + 6x$

$x^2 + 6x$ $= \left(x + \frac{6}{2}\right)^2 - \left(\frac{6}{2}\right)^2$ $= (x + 3)^2 - 9$	**1** Write $x^2 + bx + c$ in the form $\left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 + c$ **2** Simplify.

Complete the square for the expression $2x^2 - 7x$

$2x^2 - 7x$ $= 2\left(x^2 - \frac{7}{2}x\right)$ $= 2\left[\left(x - \frac{7}{4}\right)^2 - \left(\frac{7}{4}\right)^2\right]$ $= 2\left(x - \frac{7}{4}\right)^2 - \frac{49}{8}$	**1** Before completing the square write $ax^2 + bx + c$ in the form $a\left(x^2 + \frac{b}{a}x\right) + c$ **2** Now complete the square by writing $x^2 - \frac{7}{2}x$ in the form $\left(x + \frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2$ **3** Expand and Simplify

Solve $x^2 + 6x + 4 = 0$. Give your solutions in surd form.

$x^2 + 6x + 4 = 0$ $(x + 3)^2 - 9 + 4 = 0$ $(x + 3)^2 - 5 = 0$ $(x + 3)^2 = 5$ $x + 3 = \pm\sqrt{5}$ $x = \pm\sqrt{5} - 3$ So $x = -\sqrt{5} - 3$ or $x = \sqrt{5} - 3$	**1** Write $x^2 + bx + c = 0$ in the form $\left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 + c = 0$ **2** Simplify. **3** Rearrange the equation to work out x. First, add 5 to both sides. **4** Square root both sides. Remember that the square root of a value gives two answers. **5** Subtract 3 from both sides to solve the equation. **6** Write down both solutions.

Practice Questions

Set 1

Solve the following equations:

a. $x^2 - 2x - 15 = 0$ **b.** $x^2 = 9x$ **c.** $6x^2 + 13x - 5 = 0$

d. $x^2 - 5x + 18 = 2 + 3x$

Set 2

Solve $3x^2 - 7x - 1 = 0$ by using the formula.

Set 3

Complete the square for the expressions:

a. $x^2 + 8x$ **b.** $x^2 - 3x$ **c.** $2x^2 - 12x$

Set 4

Solve the equation $x^2 + 8x + 10 = 0$ by completing the square.

Give your answers in surd form.

Video Solutions

Set 1

Set 2

Set 3

Set 4

Additional Practice

Set 1

1 Solve

a $6x^2 + 4x = 0$ **b** $28x^2 - 21x = 0$

c $x^2 + 7x + 10 = 0$ **d** $x^2 - 5x + 6 = 0$

e $x^2 - 3x - 4 = 0$ **f** $x^2 + 3x - 10 = 0$

g $2x^2 - 7x - 4 = 0$ **h** $3x^2 - 13x - 10 = 0$

Set 2

1 Solve, giving your solutions in surd form.

a $3x^2 + 6x + 2 = 0$ **b** $2x^2 - 4x - 7 = 0$

2 Solve the equation $x^2 - 7x + 2 = 0$

Give your solutions in the form $\frac{a \pm \sqrt{b}}{c}$, where a, b and c are integers.

3 Solve $10x^2 + 3x + 3 = 5$

Give your solution in surd form.

Set 3

1 Complete the square for the following expressions:

a $x^2 + 8x$ **b** $x^2 - 10x$ **c** $x^2 - x$

d $3x^2 - 15x$

Set 4

1 Solve by completing the square.

a $x^2 - 4x - 3 = 0$ **b** $x^2 - 10x + 4 = 0$

c $x^2 + 8x - 5 = 0$ **d** $x^2 - 2x - 6 = 0$

Solutions

Simultaneous Equations

Worked Examples

Solve the simultaneous equations $3x + y = 5$ and $x + y = 1$

$3x + y = 5$ $-\quad x + y = 1$ $2x \quad = 4$ So $x = 2$ Using $x + y = 1$ $2 + y = 1$ So $y = -1$ Check: equation 1: $3 \times 2 + (-1) = 5$ YES equation 2: $2 + (-1) = 1$ YES	**1** Subtract the second equation from the first equation to eliminate the y term. **2** To find the value of y, substitute $x = 2$ into one of the original equations. **3** Substitute the values of x and y into both equations to check your answers.

Solve $x + 2y = 13$ and $5x - 2y = 5$ simultaneously.

$x + 2y = 13$ $+\quad 5x - 2y = 5$ $6x \quad = 18$ So $x = 3$ Using $x + 2y = 13$ $3 + 2y = 13$ So $y = 5$ Check: equation 1: $3 + 2 \times 5 = 13$ YES equation 2: $5 \times 3 - 2 \times 5 = 5$ YES	**1** Add the two equations together to eliminate the y term. **2** To find the value of y, substitute $x = 3$ into one of the original equations. **3** Substitute the values of x and y into both equations to check your answers.

Solve the simultaneous equations $y = 2x - 1$ and $y = x^2 - 4$

$2x - 1 = x^2 - 4$	**1** Substitute $2x - 1$ for y in the linear equation
$x^2 - 2x - 3 = 0$	**2** Rearrange to obtain a quadratic equation whose RHS is zero
$(x+1)(x-3) = 0$	**3** Factorize
$x = -1$ and $x = 3$	**4** Find two values for x
$x = -1,\ y = -3$ $x = 3,\ y = 5$	**5** Substitute each of these values in turn into the other equation to find two values for y
Check: Equation 1: $-3 = 2(-1) - 1$ YES $5 = 2(3) - 1$ YES Equation 2: $-1 = (-1)^2 - 4$ YES $5 = (3)^2 - 4$ YES	**6** Substitute both values for x and y into both equations to check your answers.

Solve simultaneously, $y = x + 1$ and $y = 1 + \frac{4}{x}$

$x + 1 = 1 + \frac{4}{x}$	**1** Substitute $x+1$ for y in the linear equation
$x^2 + x = x + 4$	**2** Multiply both sides by x
$x^2 - 4 = 0$	**3** Rearrange to obtain a quadratic equation whose RHS is zero
$(x-2)(x+2) = 0$	**3** Factorize
$x = 2$ and $x = -2$	**4** Find two values for x
$x = 2,\ y = 3$ $x = -2, y = -1$	**5** Substitute each of these values in turn into the other equation to find two values for y
Check: Equation 1: $3 = 2 + 1$ YES $-1 = -2 + 1$ YES Equation 2: $3 = 1 + (4 \div 2)$ YES $-1 = 1 + (4 \div (-2))$ YES	**6** Substitute both values for x and y into both equations to check your answers.

Practice Questions

Set 1

Solve the simultaneous equations:

a. $2x + 3y = 8$
$3x - y = 23$

b. $4x - 5y = 4$
$6x + 2y = 25$

Set 2

Solve the simultaneous equations:

$2x + 2y = 3$
$x^2 + 3xy = 10$

Video Solutions

Set 1

Set 2

Additional Practice

Set 1

1 $4x + y = 8$
$x + y = 5$

2 $3x + y = 7$
$3x + 2y = 5$

3 $4x + y = 3$
$3x - y = 11$

4 $3x + 4y = 7$
$x - 4y = 5$

5 $2x + y = 11$
$x - 3y = 9$

6 $2x + 3y = 11$
$3x + 2y = 4$

Set 2

Solve these simultaneous equations.

1 $xy = 9$ and $y = x$

2 $x^2 + y^2 = 50$ and $y = x$

3 $xy - 3 = 16$ and $x - 19y = 0$

Solutions

Inequalities

Worked Examples

Solve $2x - 5 < 7$

$2x - 5 < 7$ $2x < 12$ $x < 6$	**1** Add 5 to both sides. **2** Divide both sides by 2.

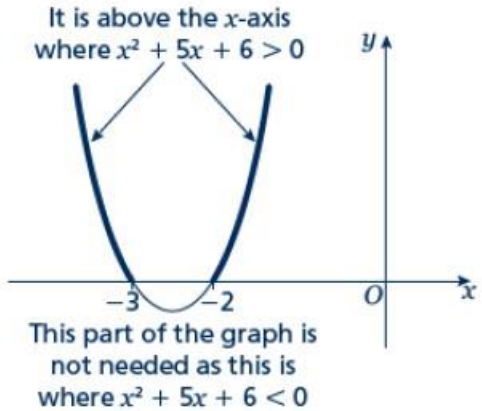
Solve $2 - 5x \geq -8$

$2 - 5x \geq -8$ $-5x \geq -10$ $x \leq 2$	**1** Subtract 2 from both sides. **2** Divide both sides by -5. Remember to reverse the inequality when dividing by a negative number.

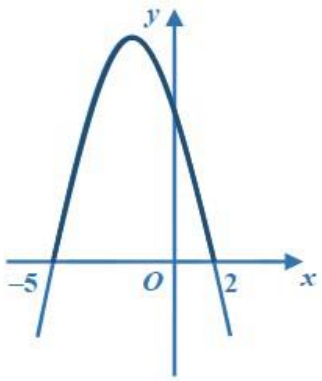
Solve $4(x - 2) > 3(9 - x)$

$4(x - 2) > 3(9 - x)$ $4x - 8 > 27 - 3x$ $7x - 8 > 27$ $7x > 35$ $x > 5$	**1** Expand the brackets. **2** Add $3x$ to both sides. **3** Add 8 to both sides. **4** Divide both sides by 7.

Find the set of values of x which satisfy $x^2 + 5x + 6 > 0$

$x^2 + 5x + 6 = 0$ $(x + 3)(x + 2) = 0$ $x = -3$ or $x = -2$  $x < -3$ or $x > -2$	**1** Solve the quadratic equation by factorising. **2** Sketch the graph of $y = (x + 3)(x + 2)$ **3** Identify on the graph where $x^2 + 5x + 6 > 0$, i.e. where $y > 0$ **4** Write down the values which satisfy the inequality $x^2 + 5x + 6 > 0$

Find the set of values of x which satisfy $-x^2 - 3x + 10 \geqslant 0$

$-x^2 - 3x + 10 = 0$ $(-x + 2)(x + 5) = 0$ $x = 2$ or $x = -5$  $-5 \leqslant x \leqslant 2$	**1** Solve the quadratic equation by factorising. **2** Sketch the graph of $y = (-x + 2)(x + 5) = 0$ **3** Identify on the graph where $-x^2 - 3x + 10 \geqslant 0$, i.e. where $y \geqslant 0$ **3** Write down the values which satisfy the inequality $-x^2 - 3x + 10 \geqslant 0$

Practice Questions

Set 1

Find the set of values of x for which:

a. $5x + 9 \geq x + 20$ **b.** $12 - 3x < 27$ **c.** $3(x - 5) > 5 - 2(x - 8)$

Set 2

Find the set of values of x for which:

$3 - 5x - 2x^2 < 0$

Video Solutions

Set 1

Set 2

Additional Practice

Set 1

1 Solve

a $2 - 4x \geq 18$ **b** $3 \leq 7x + 10 < 45$ **c** $6 - 2x \geq 4$

d $4x + 17 < 2 - x$ **e** $4 - 5x < -3x$ **f** $-4x \geq 24$

2 Solve these inequalities

a $3t + 1 < t + 6$ **b** $2(3n - 1) \geq n + 5$

3 Solve

a $3(2 - x) > 2(4 - x) + 4$ **b** $5(4 - x) > 3(5 - x) + 2$

Set 2

1 Find the set of values of x for which $(x + 7)(x - 4) \leqslant 0$

2 Find the set of values of x for which $x^2 - 4x - 12 \geqslant 0$

3 Find the set of values of x for which $2x^2 - 7x + 3 < 0$

4 Find the set of values of x for which $4x^2 + 4x - 3 > 0$

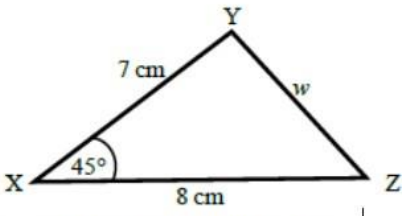
5 Find the set of values of x for which $12 + x - x^2 \geqslant 0$

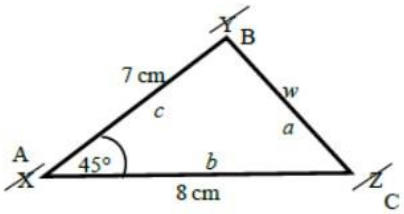
Solutions

Trigonometry

Worked Examples

Work out the length of side w.
Give your answer correct to 3 significant figures.





$a^2 = b^2 + c^2 - 2bc\cos A$

$w^2 = 8^2 + 7^2 - 2\times 8\times 7\times \cos 45°$

$w^2 = 33.804\,040\,51...$

$w = \sqrt{33.804\,040\,51}$

$w = 5.81$ cm

1 Always start by labelling the angles and sides.

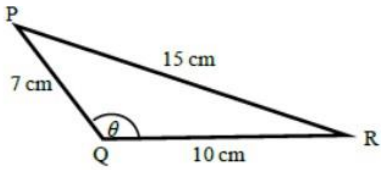
2 Write the cosine rule to find the side.

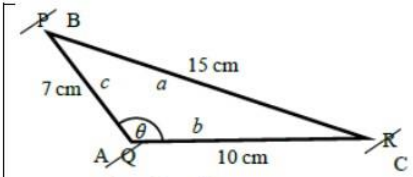
3 Substitute the values a, b and A into the formula.

4 Use a calculator to find w^2 and then w.

5 Round your final answer to 3 significant figures and write the units in your answer.

Work out the size of angle θ.
Give your answer correct to 1 decimal place.





$\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$

$\cos\theta = \dfrac{10^2 + 7^2 - 15^2}{2\times 10\times 7}$

$\cos\theta = \dfrac{-76}{140}$

$\theta = 122.878\,349...$

$\theta = 122.9°$

1 Always start by labelling the angles and sides.

2 Write the cosine rule to find the angle.

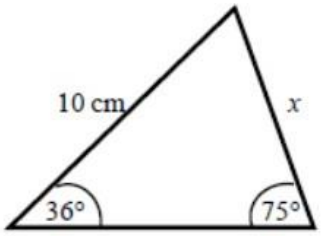
3 Substitute the values a, b and c into the formula.

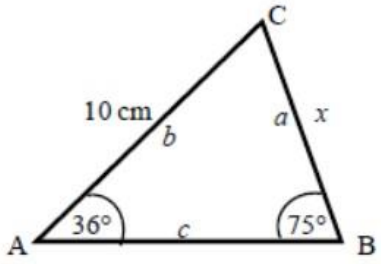
4 Use $\cos^{-1}$ to find the angle.

5 Use your calculator to work out $\cos^{-1}(-76 \div 140)$.

6 Round your answer to 1 decimal place and write the units in your answer.

Work out the length of side x.
Give your answer correct to 3 significant figures.





$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\frac{x}{\sin 36°} = \frac{10}{\sin 75°}$$

$$x = \frac{10 \times \sin 36°}{\sin 75°}$$

$x = 6.09$ cm

1 Always start by labelling the angles and sides.

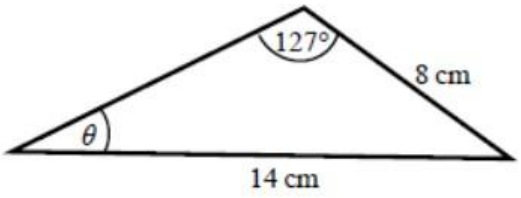
2 Write the sine rule to find the side.

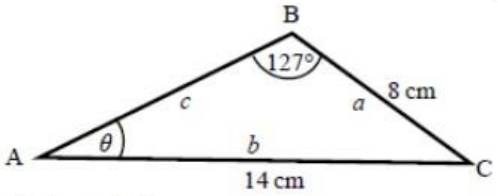
3 Substitute the values a, b, A and B into the formula.

4 Rearrange to make x the subject.

5 Round your answer to 3 significant figures and write the units in your answer.

Work out the size of angle θ.
Give your answer correct to 1 decimal place.





$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\frac{\sin \theta}{8} = \frac{\sin 127°}{14}$$

$$\sin \theta = \frac{8 \times \sin 127°}{14}$$

$\theta = 27.2°$

1 Always start by labelling the angles and sides.

2 Write the sine rule to find the angle.

3 Substitute the values a, b, A and B into the formula.

4 Rearrange to make $\sin \theta$ the subject.

5 Use $\sin^{-1}$ to find the angle. Round your answer to 1 decimal place and write the units in your answer.

Practice Questions

Set 1

Calculate the length of the side AB of the triangle ABC in which $AC = 6.5$ cm, $BC = 8.7$ cm and $\angle ACB = 100°$.

Set 2

Find the size of the smallest angle in a triangle whose sides have lengths 3 cm, 5 cm and 6 cm.

Set 3

In $\triangle ABC$, $AB = 8$ cm, $\angle BAC = 30°$ and $\angle BCA = 40°$. Find BC.

Set 4

In $\triangle ABC$, $AB = 3.8$ cm, $BC = 5.2$ cm and $\angle BAC = 35°$. Find $\angle ABC$.

Set 5

a. Sketch the graph of $y = \cos\theta$ in the interval $-360° \leq \theta \leq 360°$.

b. **i.** Sketch the graph of $y = \sin x$ in the interval $-180° \leq x \leq 270°$

ii. $\sin(-30°) = -0.5$. Use your graph to determine two further values of x for which $\sin x = -0.5$.

Video Solutions

Set 1

Set 2

Set 3

Set 4

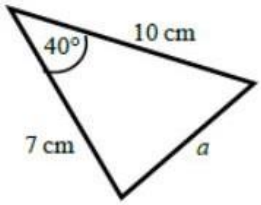
Set 5

Additional Practice

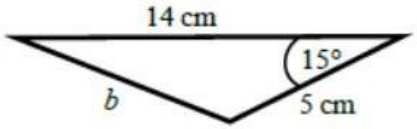
Set 1

1 Work out the length of the unknown side in each triangle.
Give your answers correct to 3 significant figures.

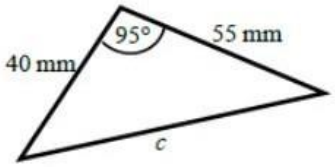
a



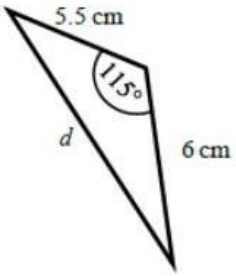
b



c



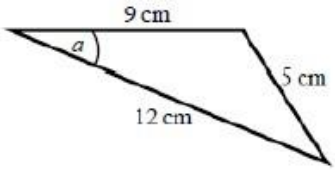
d



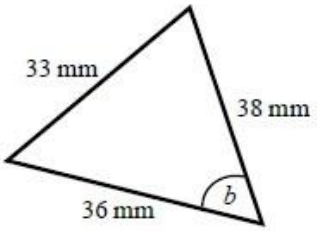
Set 2

1 Calculate the angles labelled θ in each triangle.
Give your answer correct to 1 decimal place.

a



b



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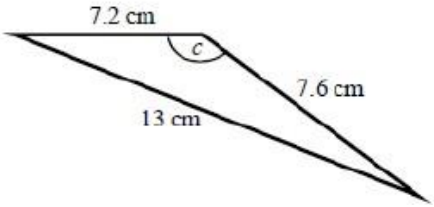
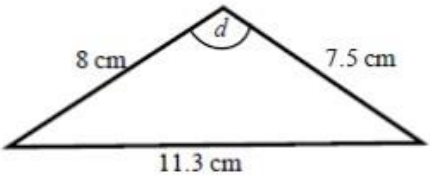


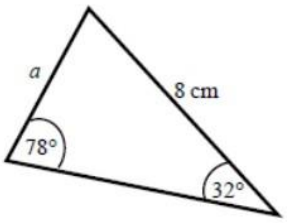
d



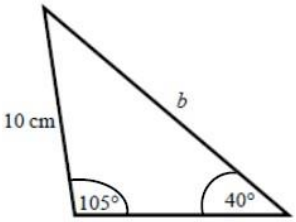
Set 3

1 Find the length of the unknown side in each triangle.
Give your answers correct to 3 significant figures.

a



b



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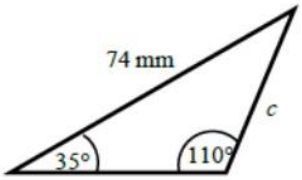
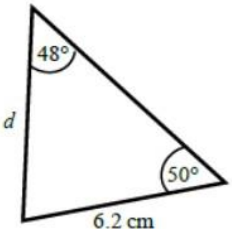


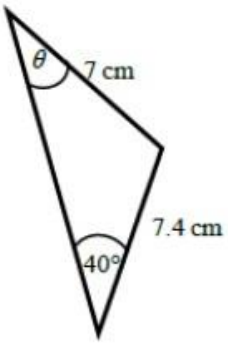
d



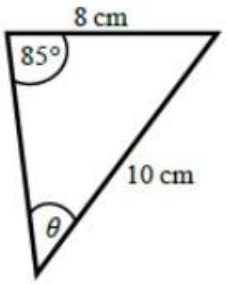
Set 4

1 Calculate the angles labelled θ in each triangle.
Give your answer correct to 1 decimal place.

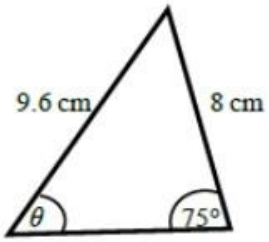
a



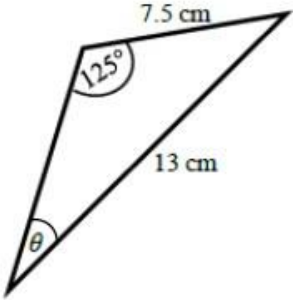
b



c

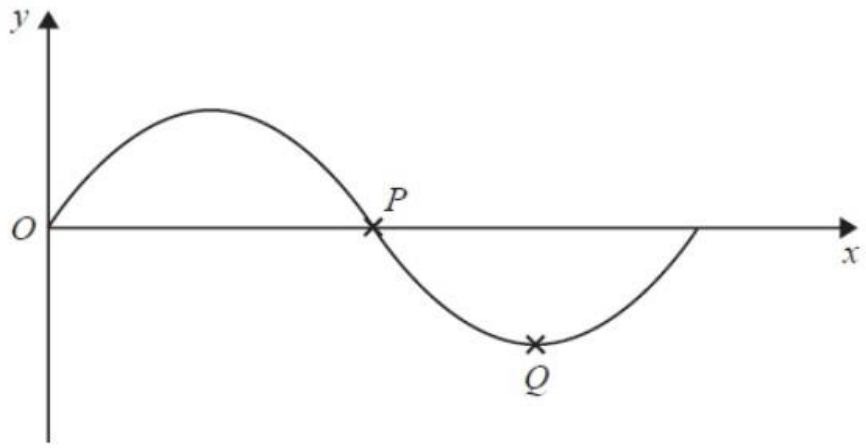


d



Set 5

1 The diagram shows part of a sketch of the curve $y = \sin x°$



(a) Write down the coordinates of

(i) the point P

(..... , ...)

(ii) the point Q

(..... , ...)

(b) Sketch the graph of $y = \tan x$ for $0° \leq x \leq 360°$

Show the coordinates of any points of intersection with the coordinate axes.

Solutions

Functions

What is a Function? *In maths, a function is something that takes an input and produces an output. Functions can be given as function machines or as mathematical expressions. Sometimes, functions are displayed in the form* $f: x \rightarrow 3x - 5$, *which is the same as* $f(x) = 3x - 5$.

Worked Examples

Type 1: *Evaluating Functions*

Example: *A function is given by* $f(x) = 3x + 1$. *Find* $f(10)$.

To solve this, replace x with 10 *and calculate the result.*

$Wf(10) = 3 \times 10 + 1 = 31.$

Type 2: *Composite Functions*

Example: *Let* $f(x) = 2x - 3$ *and* $g(x) = x + 1$. *Find* $fg(x)$.

To find $fg(x)$, *replace x in* $f(x)$ *with* $g(x)$. $fg(x) = f\big(g(x)\big) = 2(x + 1) - 3.$

Expand the brackets and simplify: $2(x + 1) - 3 = 2x + 2 - 3$

$= 2x - 1.$

Example 1: *Composite Functions (Additional) Let* $f(x) = x - 3$ *and* $g(x) = x^2$

. *a) Find* $fg(10)$: *First find* $g(10)$, *then apply* $f(x)$ *to the answer.*

$g(10) = 10^2 = 100.$

$So, fg(10) = f(100) = 100 - 3 = 97.$

b) Find $gf(-4)$: *First find* $f(-4)$, *then apply* $g(x)$ *to the answer.*

$f(-4) = -4 - 3 = -7. So, gf(-4) = g(-7) = (-7)^2 = 49.$

c) Find an expression for $fg(x)$:

Input $g(x)$ *into* $f(x)$. $fg(x) = f(g(x)) = g(x) - 3 = x\text{^}2 - 3.$

Type 3: *Inverse Functions*

An inverse function is a function acting in reverse.

The inverse function of $f(x)$ *is given by* $f^{-1}(x)$, *and it tells us how to go from an output of* $f(x)$ *back to its input.*

Example: *Given that* $f(x) = \frac{x + 8}{3}$, *find* $f^{-1}(x)$.

Step 1: *Write the equation in the form* $x = f(y)$.

Replace all $x's$ *with* $y's$ *and set the equation equal to* x. $f(x)$

$= \frac{x + 8}{3}$ *becomes* $x = \frac{y + 8}{3}$.

Step 2: *Rearrange the equation to make y the subject.* $x = (y + 8)/3\ 3x$

$= y + 8\ 3x - 8 = y.$ *Step* 3: *Replace y with* $f^{-1}(x)$. y

$= 3x - 8\ f^{-1}(x) = 3x - 8.$

Example 2: *Inverse Functions (Additional)*

Given that $f(x) = 3x - 9$, *find* $f^{-1}(x)$.

Step 1: *Write the equation in the form* $x = f(y)$.

$f(x) = 3x - 9$ *becomes* $x = 3y - 9$.

Step 2: *Rearrange to make* y *the subject.* $x = 3y - 9$

$x + 9 = 3y$

$\frac{x + 9}{3} = y.$

Step 3: *Replace* y *with* $f^{-1}(x)$.

$\frac{x + 9}{3} = y$

$f^{-1}(x) = \frac{x+9}{3}$.

Question 1: *Let* $f(x) = \frac{10}{3x-5}$.

$a)$ *Find* $f(10)$

$.b)$ *Find* $f(2)$.

$c)$ *Find* $f(-1)$.

Question 2: *Let* $f(x) = \frac{15}{x}$ *and* $g(x) = 2x - 5$.

$a)$ *Find* $fg(4)$

$.b)$ *Find* $gf(-30)$.

$c)$ *Find* $gf(x)$.

Question 3: *Find the inverse function of* $f(x) = \frac{5}{x-4}$.

Question 4: *Find the inverse function of* $g(x) = \frac{4}{x} + 3$.

Solutions

Solutions

Practice Assessment

1. Simplify:
   a) $2x^5$
   b) x^4
   c) $3x^3\ y^2$
   d) $72x^{\frac{17}{3}}$
2. $x = \pm 2$
3. $x = 4$
4.
   a) $6\sqrt{10}$
   b) $-9 + 7\sqrt{3}$
   c) $-9 + 7\sqrt{3}$
5. $27x^3 + 54x^2 + 36x + 8$
6.
   a) $(x + 7)(x - 7)$
   b) $(x + 5)(x + 3)$
   c) $(2x + 3)(3x - 2)$
7.
   a) $x = -8, x = 5$
   b) $x = 1, x = 3$
8.
   a) $x = 3, y = 8$
   b) $x = \frac{11}{9}, y = -\frac{17}{9}$
   c) $x = 3, y = 1$
      $x = -1, y = 5$
9.
   a) $x \leq 3$
   b) $x \geq 3$
   c) $x < 3, x > 7$
10.
   a) 10
   b) $4x^2 + 2$
   c) 18

Indices

Additional Practice

Set 1

1. (a) a^9

(b) $9e^5f^6$

(c) 3

2. (a) x^{10}

(b) m^{12}

(c) $3a^{-4}f^6$

Set 2

1 **a** $\frac{3x^3}{2}$ **b** $5x^2$

c $3x$ **d** $\frac{y}{2x^2}$

e $y^{\frac{1}{2}}$ **f** c^{-3}

g $2x^6$ **h** x

Set 3

1 **a** 7 **b** 4

2 **a** 125 **b** 32

3 **a** $\frac{1}{25}$ **b** $\frac{1}{64}$

4 **a** $\frac{1}{2}$ **b** $\frac{1}{9}$

Surds

Additional Practice

Set 1

1 **a** $3\sqrt{5}$ **b** $5\sqrt{5}$

c $4\sqrt{3}$ **d** $5\sqrt{7}$

2 **a** $15\sqrt{2}$ **b** $\sqrt{5}$

c $3\sqrt{2}$ **d** $\sqrt{3}$

e $6\sqrt{7}$ **f** $5\sqrt{3}$

Set 2

1 **a** -1 **b** $9-\sqrt{3}$

c $10\sqrt{5}-7$ **d** $26-4\sqrt{2}$

2 $x-y$

3 18

4 $1+2\sqrt{2}$

Set 3

1 **a** $\frac{\sqrt{5}}{5}$ **b** $\frac{\sqrt{11}}{11}$

c $\frac{2\sqrt{7}}{7}$ **d** $\frac{\sqrt{2}}{2}$

e $\sqrt{2}$ **f** $\sqrt{5}$

2 **a** $\frac{3+\sqrt{5}}{4}$ **b** $\frac{2(4-\sqrt{3})}{13}$ **c** $\frac{6(5+\sqrt{2})}{23}$

3 **a** $3+2\sqrt{2}$ **b** $\frac{\sqrt{x}+\sqrt{y}}{x-y}$

Algebraic Methods

Additional Practice

Set 1

1 **a** $\frac{2(x+2)}{x-1}$ **b** $\frac{x}{x-1}$

c $\frac{x+2}{x}$ **d** $\frac{x}{x+5}$

e $\frac{x+3}{x}$ **f** $\frac{x}{x-5}$

2 **a** $\frac{3x+4}{x+7}$ **b** $\frac{2x+3}{3x-2}$

Quadratic Equations

Additional Practice

Set 1

1 **a** $x = 0$ or $x = -\frac{2}{3}$ **b** $x = 0$ or $x = \frac{3}{4}$

c $x = -5$ or $x = -2$ **d** $x = 2$ or $x = 3$

e $x = -1$ or $x = 4$ **f** $x = -5$ or $x = 2$

g $x = -\frac{1}{2}$ or $x = 4$ **h** $x = -\frac{2}{3}$ or $x = 5$

Set 2

1 **a** $x = -1 + \frac{\sqrt{3}}{3}$ or $x = -1 - \frac{\sqrt{3}}{3}$ **b** $x = 1 + \frac{3\sqrt{2}}{2}$ or $x = 1 - \frac{3\sqrt{2}}{2}$

2 $x = \frac{7 + \sqrt{41}}{2}$ or $x = \frac{7 - \sqrt{41}}{2}$

3 $x = \frac{-3 + \sqrt{89}}{20}$ or $x = \frac{-3 - \sqrt{89}}{20}$

Set 3

1

a $(x+4)^2 - 16$ **b** $(x-5)^2 - 25$ **c** $\left(x - \frac{1}{2}\right)^2 - \frac{1}{4}$

d $3\left(x - \frac{5}{2}\right)^2 - \frac{75}{4}$ **e** $-2(x-3)^2 + 18$

Set 4

1 **a** $x = 2 + \sqrt{7}$ or $x = 2 - \sqrt{7}$ **b** $x = 5 + \sqrt{21}$ or $x = 5 - \sqrt{21}$

c $x = -4 + \sqrt{21}$ or $x = -4 - \sqrt{21}$ **d** $x = 1 + \sqrt{7}$ or $x = 1 - \sqrt{7}$

Simultaneous Equations

Additional Practice

Set 1

1 $x = 1, y = 4$ **4** $x = 3, y = -\frac{1}{2}$

2 $x = 3, y = -2$ **5** $x = 6, y = -1$

3 $x = 2, y = -5$ **6** $x = -2, y = 5$

Set 2

1 $x = -3,\ y = -3$ and $x = 3,\ y = 3$

2 $x = -5,\ y = -5$ and $x = 5,\ y = 5$

3 $x = -19,\ y = -1$ and $x = 19,\ y = 1$

Inequalities

Additional Practice

Set 1

1 **a** $x \leq -4$ **b** $-1 \leq x < 5$ **c** $x \leq 1$

d $x < -3$ **e** $x > 2$ **f** $x \leq -6$

2 **a** $t < \frac{5}{2}$ **b** $n \geq \frac{7}{5}$

3

a $x < -6$ **b** $x < \frac{3}{2}$

Set 2

1 $-7 \leqslant x \leqslant 4$

2 $x \leqslant -2$ or $x \geqslant 6$

3 $\frac{1}{2} < x < 3$

4 $x < -\frac{3}{2}$ or $x > \frac{1}{2}$

5 $-3 \leqslant x \leqslant 4$

Trigonometry

Additional Practice

Set 1

1 **a** 6.46 cm **b** 9.26 cm **c** 70.8 mm **d** 9.70 cm

Set 2

1 **a** 22.2° **b** 52.9° **c** 122.9° **d** 93.6°

Set 3

1 **a** 4.33 cm **b** 15.0 cm **c** 45.2 mm **d** 6.39 cm

Set 4

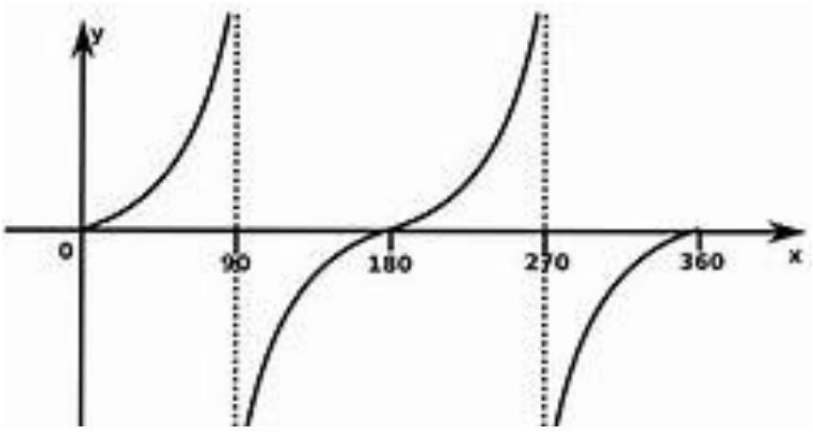
1 **a** 42.8° **b** 52.8° **c** 53.6° **d** 28.2°

Set 5

1 (a)(i) (180, 0)

(ii) (270, − 1)

(b)



Functions

Question 1: $a)\ 2/5n = 0.4$

$b)\ 10.$

$c)\ -\frac{5}{4} = -1.25.$

Question 2: $a)\ 5.$

$b)\ -6.$

$c)\ \frac{30}{x} - 5.$

Question 3: $f^{-1}(x) = \frac{5}{x} + 4.$

Question 4: $g^{-1}(x) = \frac{4}{x-3}.$